MIMO Techniques

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MIMO Classification

CSI: channel state information

MIMO
multiple input multiple output

Beamforming based MIMO
only best subchannel

- lower complexity
- long term channel statistics stable
- optimum for small SNR and interference

Multiplex Transmission
several subchannels

CSI at Tx

Waterfilling

- optimal MIMO system
- instantanious CSI
- beamforming for each subchannel

no CSI at Tx

equal Tx power/antenna
BLAST
Space Time Codes

- high capacity
- not too complex
- already realizable

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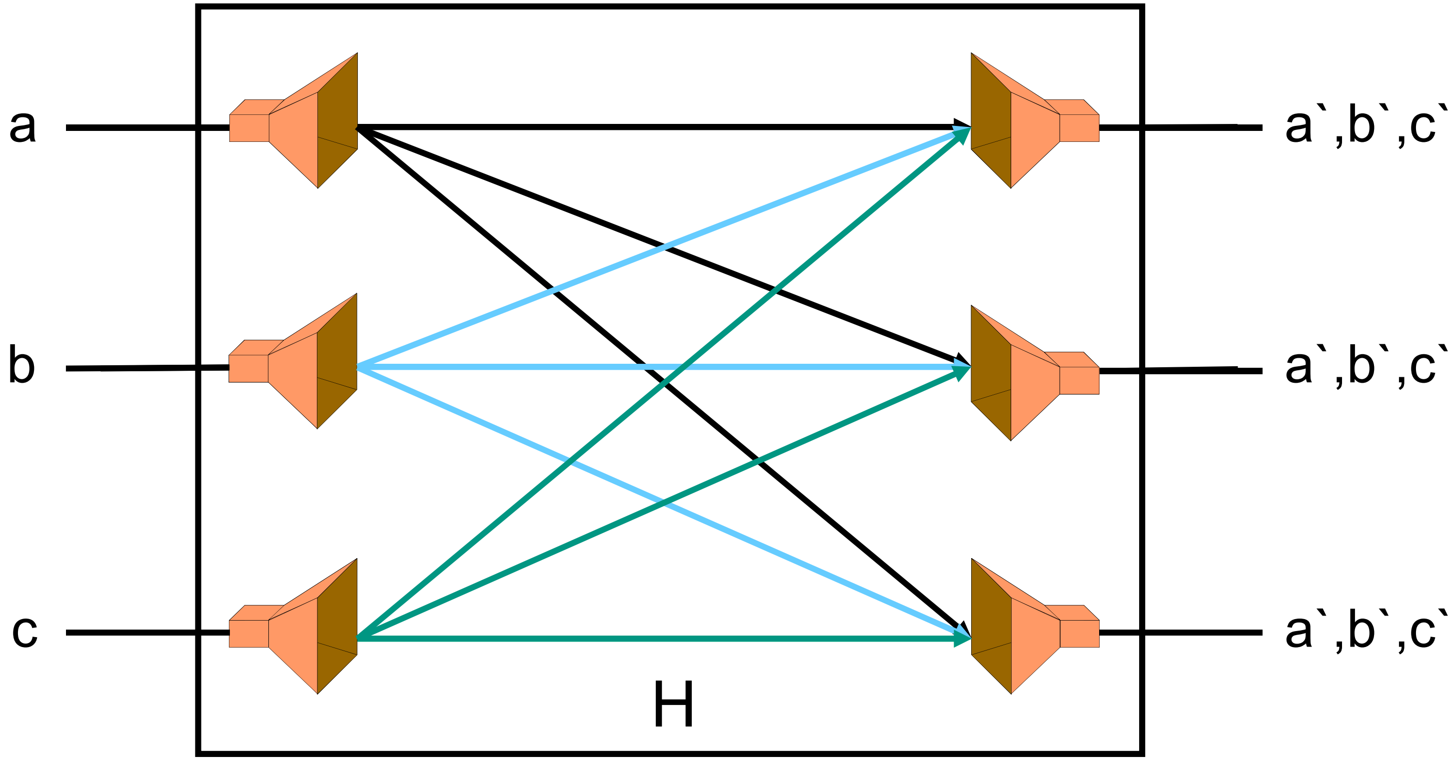
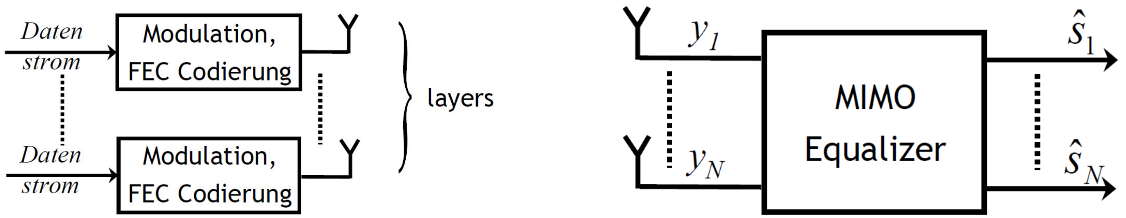
MIMO without CSI



„interference“ or „crosstalk“ among the signals

BLAST-Algorithmus

- MIMO für den Fall, dass der Kanal dem Sender nicht bekannt ist
  → Sender-Beamforming **V** und Waterfilling sind nicht möglich



- **Bell Labs Layered Space Time (BLAST)** – Verfahren
  - “Space” heißt, die Kanalkodierung geschieht über mehrere räumlich getrennte (am besten gut dekorrelierte) Sendeantennen
  - “Time” heißt, die Kanalkodierung geschieht außerdem zusätzlich über mehrere zeitlich aufeinanderfolgende Symbole (Bits)
- Entzerrung der überlagerten Datenströme im Empfänger ist notwendig
  → Für die Entzerrung muss der Kanal **H** am Empfänger geschätzt werden
- Entzerreralgorithmen für den Empfänger:
  - Lineare Entzerrer: Zero-Forcing (ZF)
    Minimum Mean Square Error (MMSE)
  - Nichtlineare Entzerrer: Successive Interference Cancellation (SIC)
    Maximum – Likelihood Detektion (ML)

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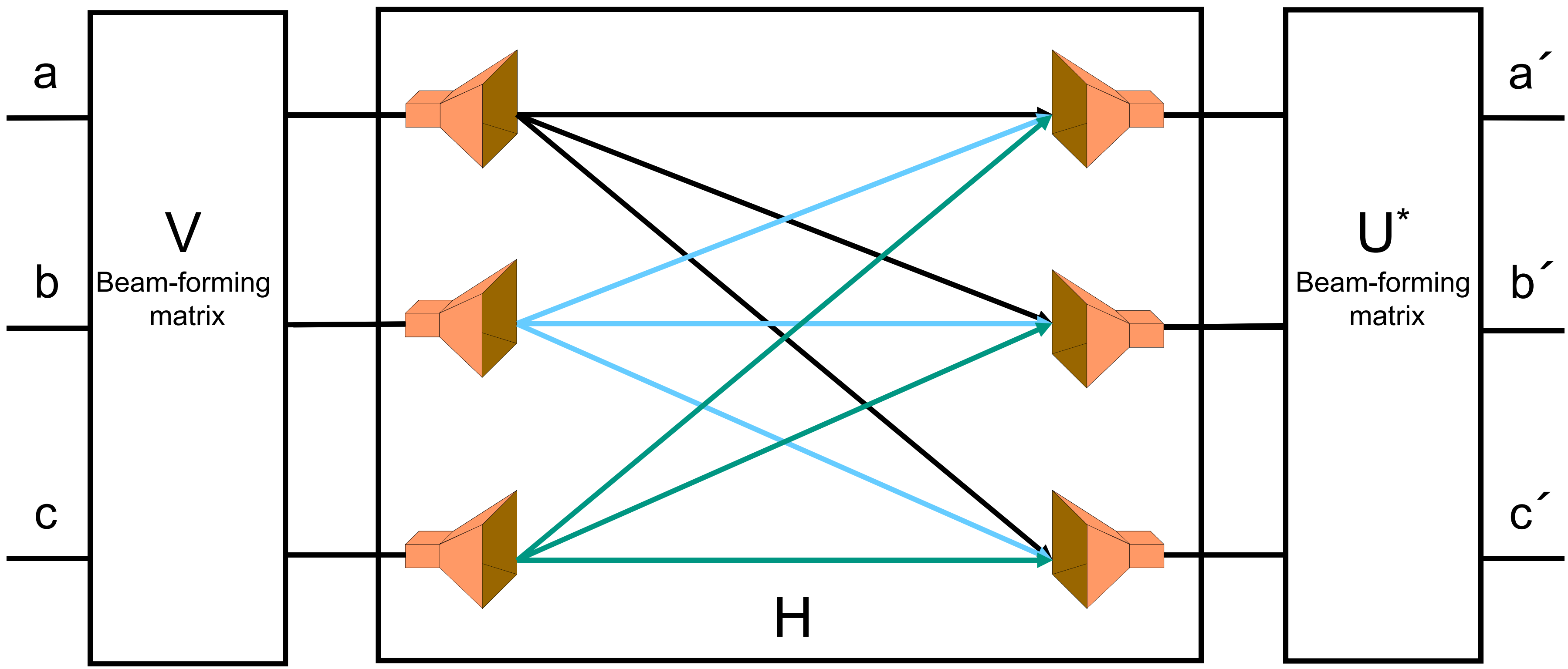

„interference“ or „crosstalk“ among the signals

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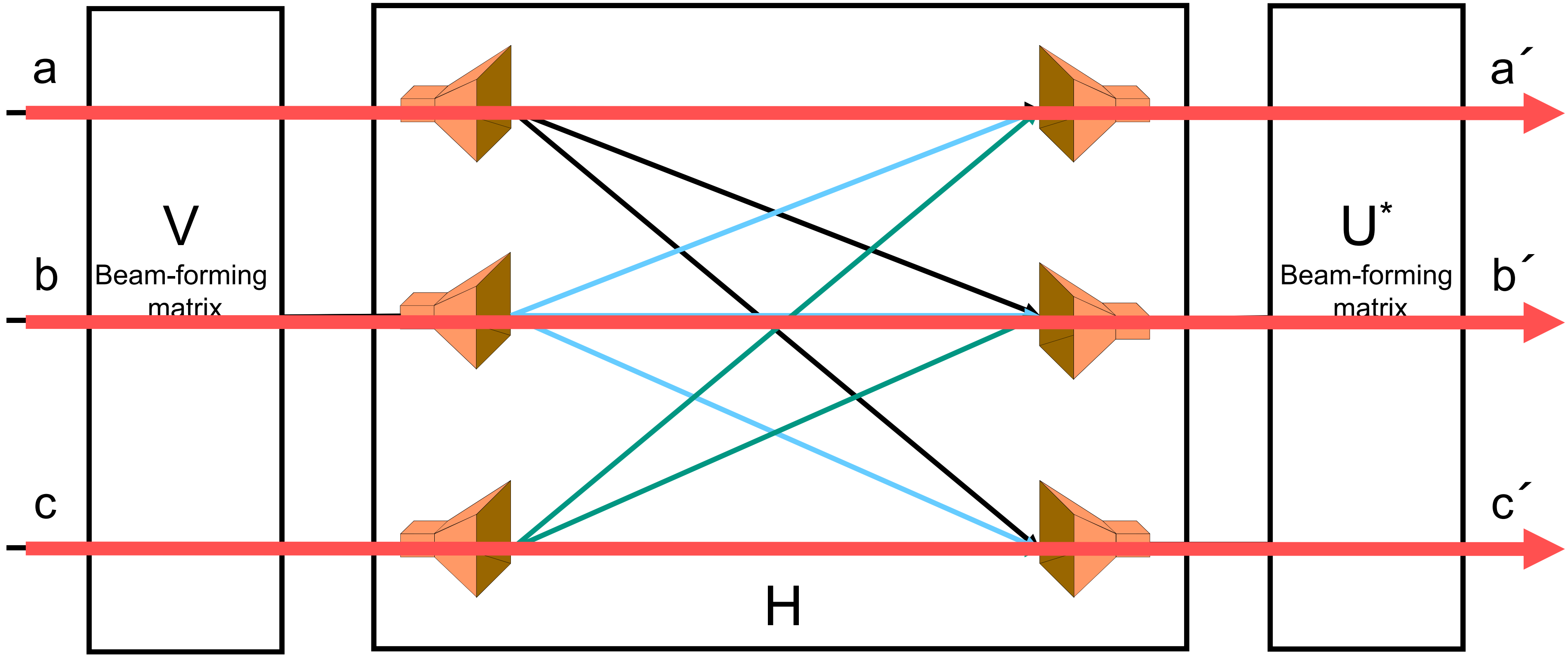
MIMO with CSI



➡ Singular Value Decomposition of **H**: spatially orthogonal sub-channels

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Singular-Value Decomposition (SVD) of H

- Further insight into the behaviour of a MIMO system: singular-value decomposition (SVD) of the channel matrix H
- Singular-value decomposition is similar to eigenvalue decomposition of a matrix but exists also for rectangular matrices
- In the following, we derive the SVD starting with the basic channel model

$$H = U \cdot S \cdot V^{*} \rightarrow S = U^{*} \cdot H \cdot V$$

$$H \in [n \times m]$$

$$S = \begin{pmatrix} \sqrt{\lambda_1} & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \sqrt{\lambda_2} & 0 & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & \ddots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \sqrt{\lambda_P} & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & 0 & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{pmatrix}$$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_P$$

$$P = \mathrm{rank}(H)$$

$$\text{Singularvalues of } H = \sqrt{\lambda_p}$$

$$\sqrt{\lambda_p} = \text{Voltage Transmission Coefficient}$$

$$\text{Eigenvalues of } HH^{*} \text{ or } H^{*}H \text{ respectively} = \lambda_p$$

$$\lambda_p = \text{Power Transmission Coefficient}$$

The goal is to transmit over the diagonal matrix S

(similar to the wired MIMO system without mutual coupling)

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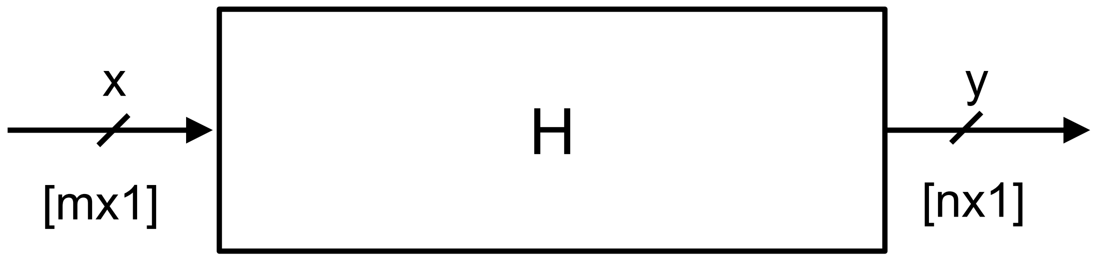
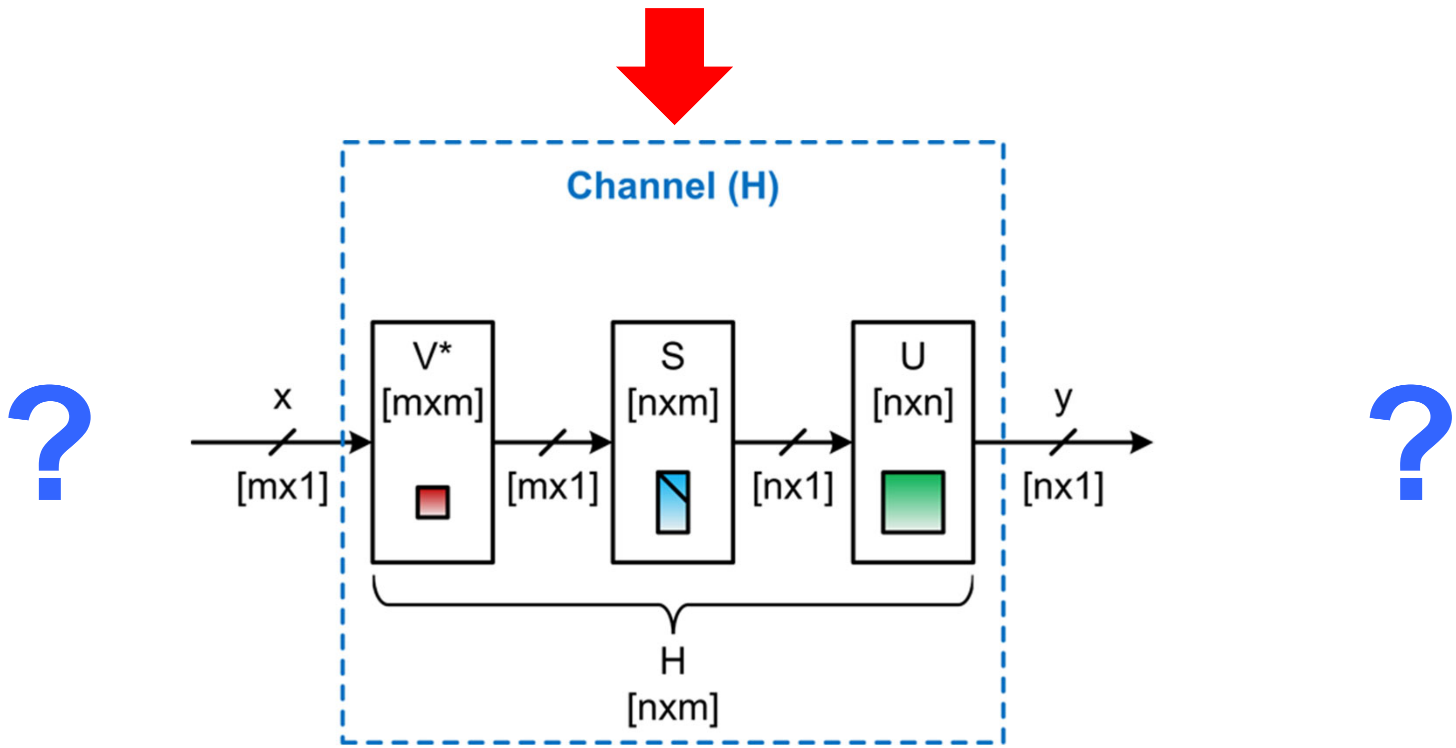
Singular-Value Decomposition (SVD) of H

- Channel Matrix H is decomposed into three matrices





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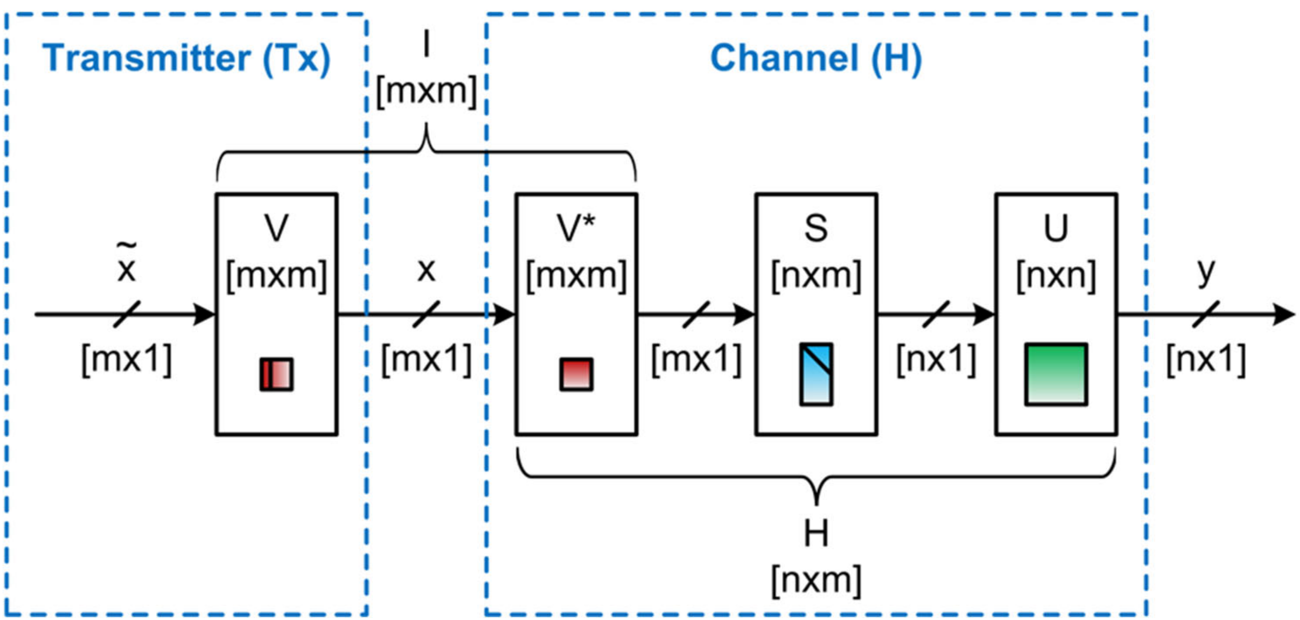
Singular-Value Decomposition (SVD) of H

- V = transmit subchannel beamforming matrix
- Each subchannel gets its own Tx-beamforming
- Compensates the Tx-part of decomposed H
- Channel State Information (CSI) at Transmitter is required
- V is an unitary matrix

$$V^* = V^{-1} \Rightarrow V^* \cdot V = I_m$$

$$I = Idenity - Matrix$$



?

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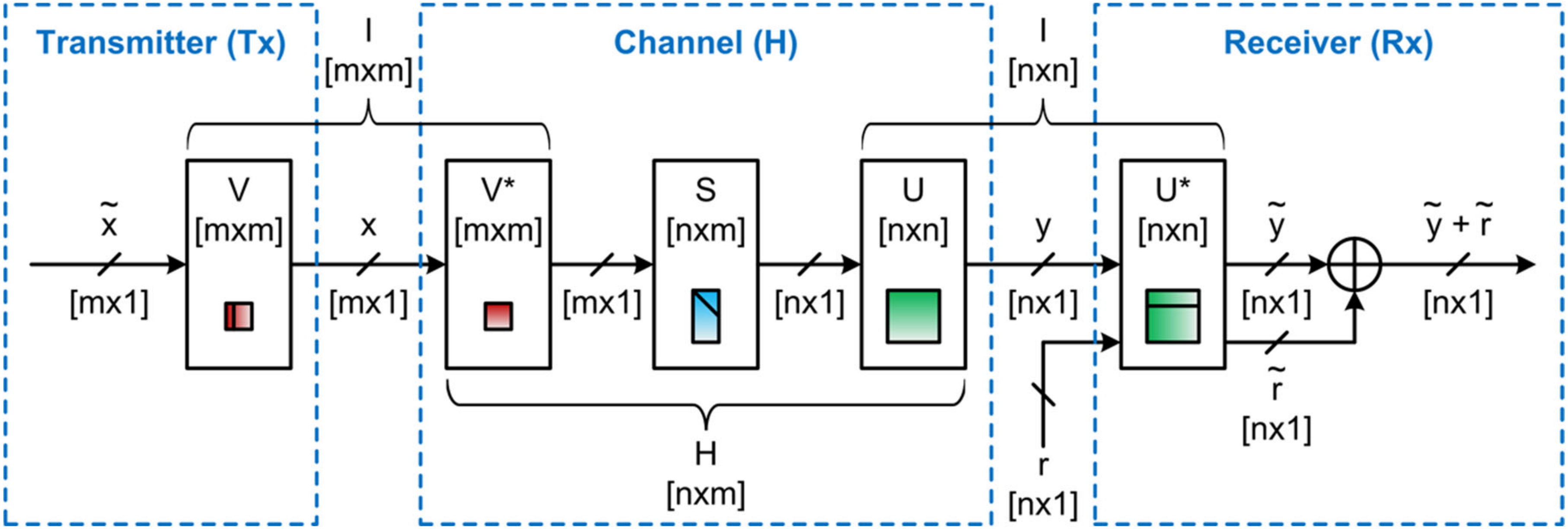
Singular-Value Decomposition (SVD) of H

- U* = receive subchannel beamforming matrix
- Each subchannel gets its own Rx-beamforming
- Compensates the Rx-part of decomposed H
- Channel State Information (CSI) at Receiver is required
- U is an unitary matrix

$$U^* = U^{-1} \Rightarrow U^* \cdot U = I_n$$

$$I = Idenity - Matrix$$



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System Equation and Signal Transformation

- The diagonal matrix S is the resulting transmission matrix
- rank(H) = number of independent (decorrelated) SISO-links that can be established

System:

$$H = U \cdot S \cdot V^*$$

$$y = H \cdot x = USV^* \cdot x$$

$$\tilde{y} = U^* USV^* V \cdot \tilde{x}$$

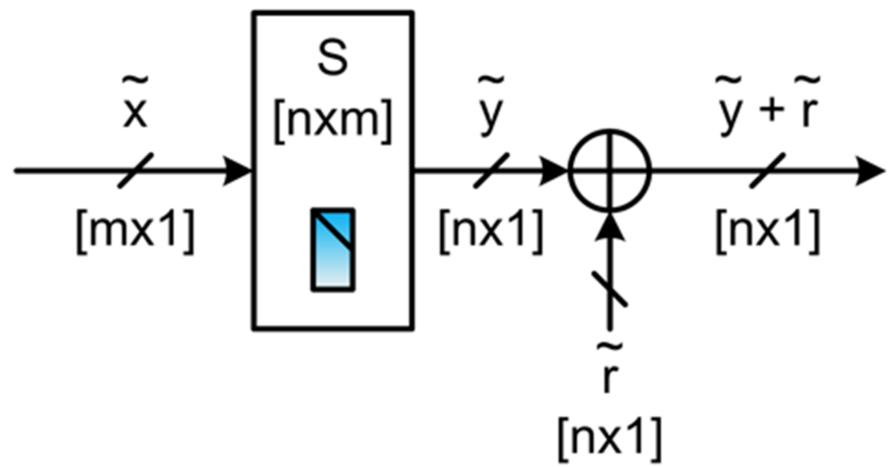
Signal-Transformation:

$$\tilde{x} = V^* \cdot x = Tx-Signal$$

$$\tilde{y} = U^* \cdot y = Rx-Signal$$

$$\tilde{r} = U^* \cdot r = Noise$$

$$\tilde{y} = S \cdot \tilde{x}$$



Beamforming Matrices in SVD

- The Matrices U and V are unitary (square) matrices

Tx-Subchannel-Beamforming-Matrix:

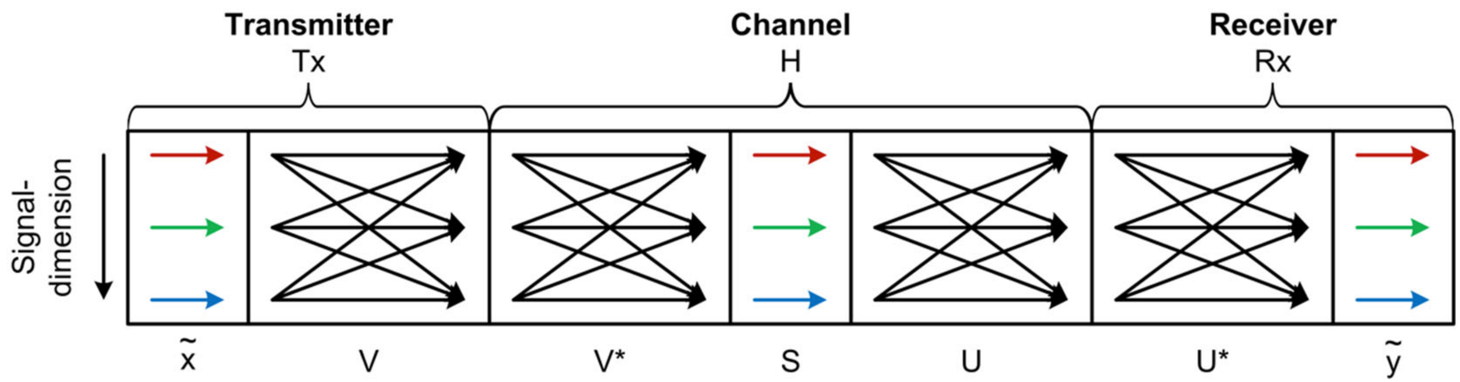
$$V^{(*)} = \begin{pmatrix} v_{11}^{(*)} & v_{12}^{(*)} & \cdots & v_{1M}^{(*)} \\ v_{21}^{(*)} & v_{22}^{(*)} & \cdots & v_{2M}^{(*)} \\ \vdots & \vdots & \ddots & \vdots \\ v_{M1}^{(*)} & v_{M2}^{(*)} & \cdots & v_{MM}^{(*)} \end{pmatrix}$$

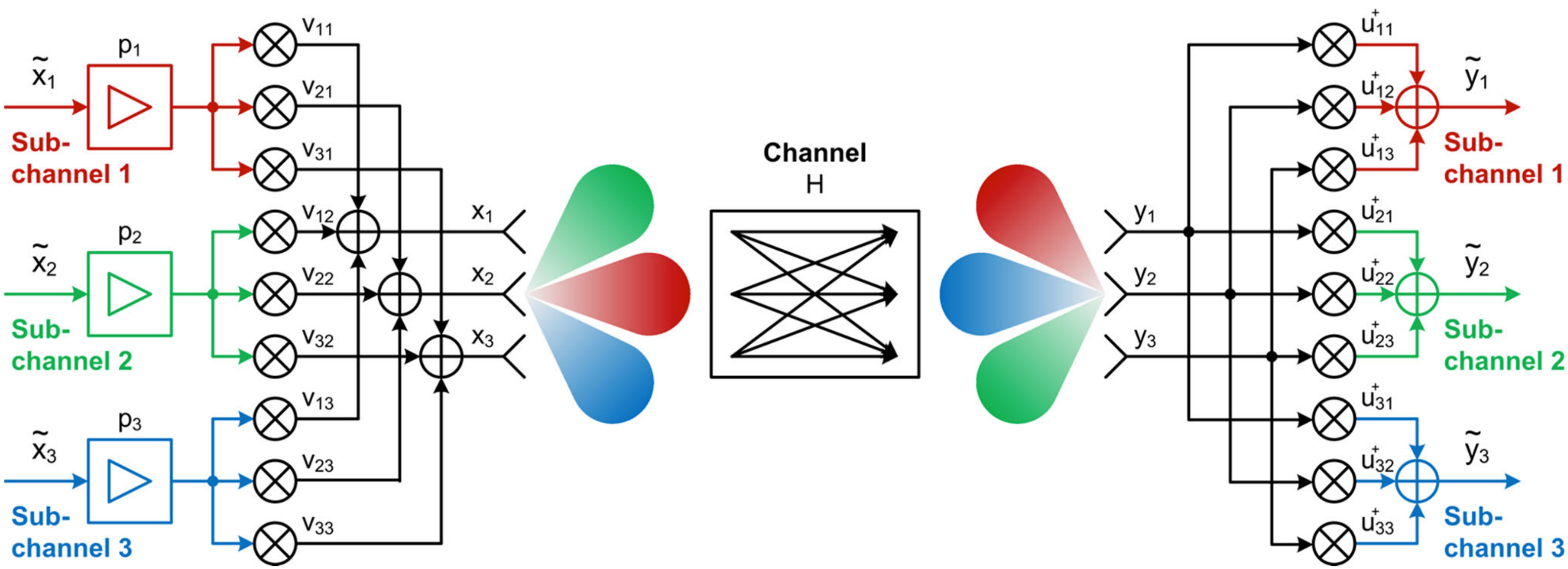
Rx-Subchannel-Beamforming-Matrix:

$$U^{(*)} = \begin{pmatrix} u_{11}^{(*)} & u_{12}^{(*)} & \cdots & u_{1N}^{(*)} \\ u_{21}^{(*)} & u_{22}^{(*)} & \cdots & u_{2N}^{(*)} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N1}^{(*)} & u_{N2}^{(*)} & \cdots & u_{NN}^{(*)} \end{pmatrix}$$

- Practical matrix dimensions for symmetrical (M=N) and unsymmetrical (M≠N) MIMO-systems?

Matrices and Signalling for M=N=3





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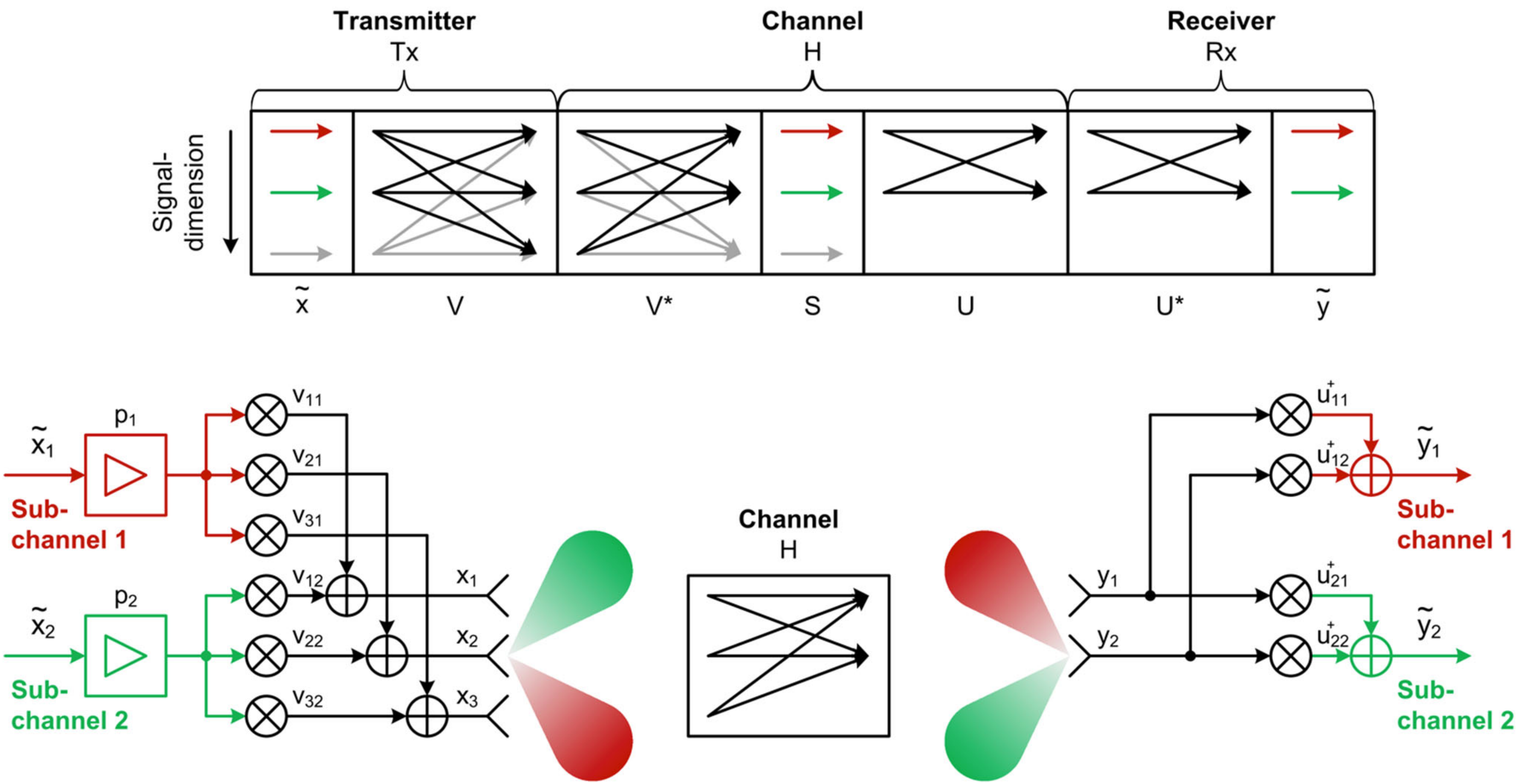
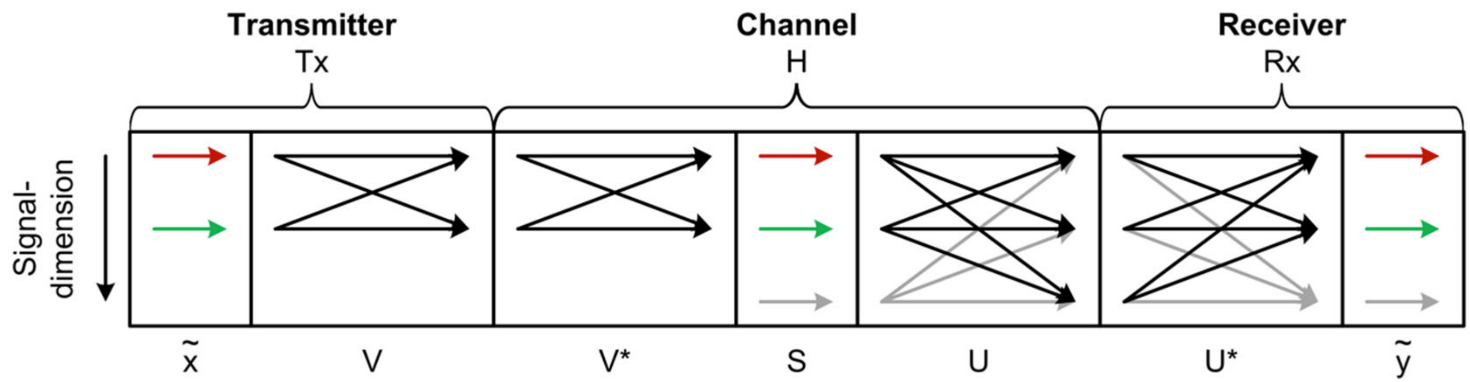
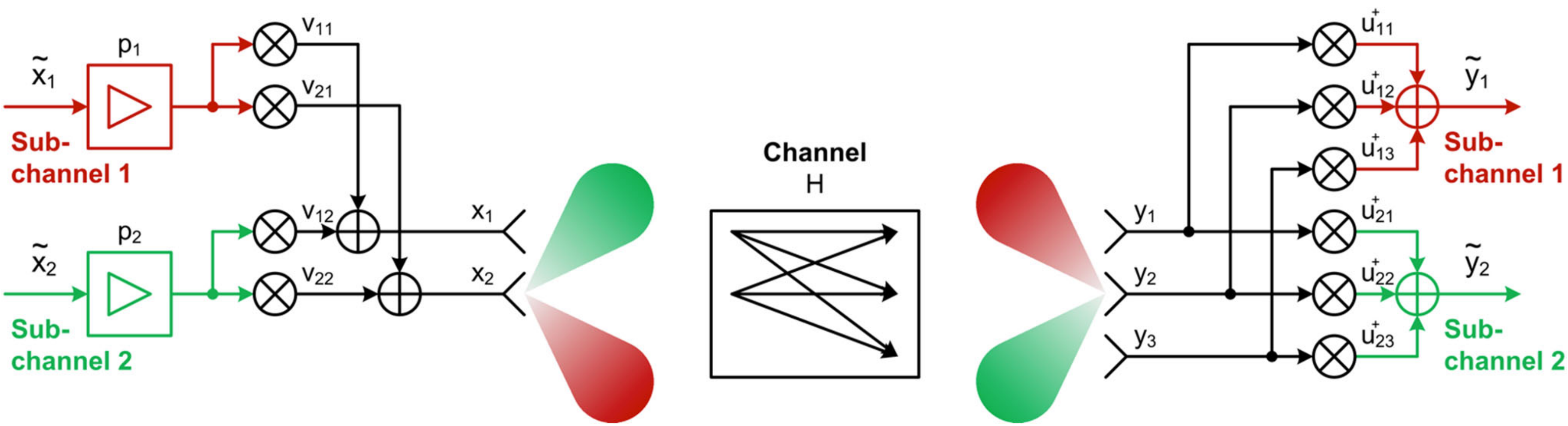
Practical Matrix Dimensions for M=3 ≠ N=2



Practical Matrix Dimensions for M=2 ≠ N=3





Normalization in SVD

- Frobenius norm
  - The squared Frobenius norm of H is given by

$$\|\mathbf{H}\|_F^2 = \sum_{n=1}^{N} \sum_{m=1}^{M} |H(n,m)|^2 = \sum_{k=1}^{\min(N,M)} \lambda_k$$

and represents the total energy of the channel

  - Often used to normalize and compare MIMO measurements. Only usable for MIMO systems.

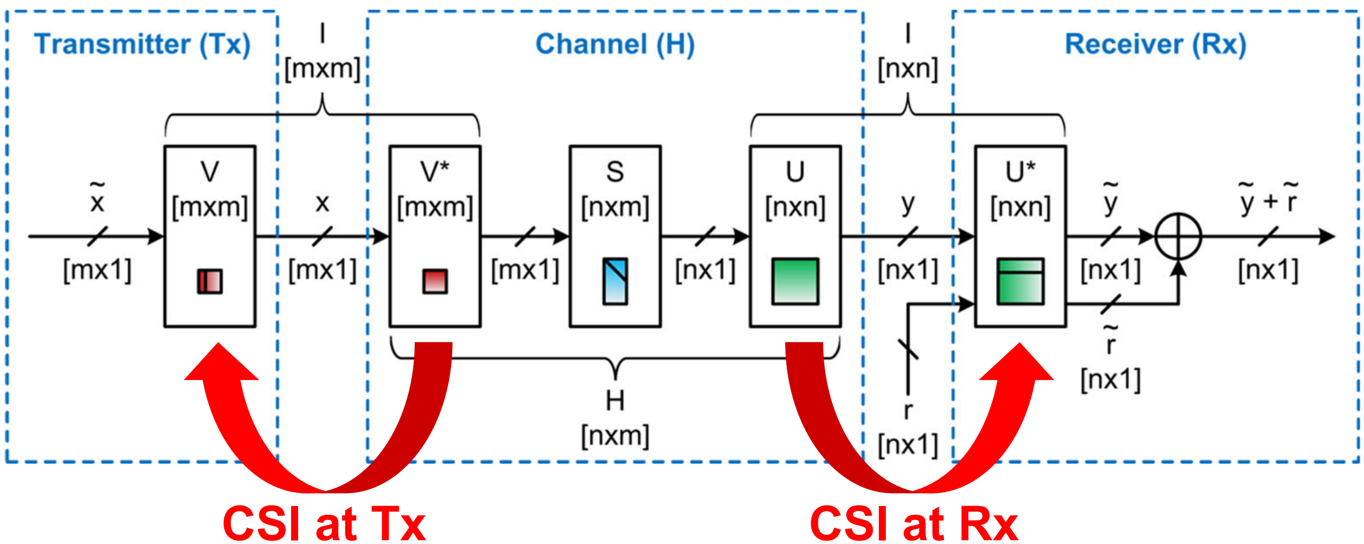
  - Fluctuations of the total channel energy are removed from the measurement data.

$$\lambda_{norm,i} = \frac{\lambda_i}{\|\mathbf{H}\|_F^2}$$

Inanoglu, Hakan, "Multiple-Input Multiple-Output System Capacity: Antenna and Propagation Aspects“, Antennas and Propagation Magazine, IEEE , vol.55, no.1, pp.253,273, Feb. 2013, doi: 10.1109/MAP.2013.6474541

Application of SVD

- The SVD of the channel matrix H has transformed the MIMO wireless link into P virtual subchannels
- The virtual subchannels are all decoupled from each other.
  They constitute a parallel set of P single-input single-output (SISO) systems
  Each subchannel is described by a scalar input-output relation λ.
- These subchannels are orthogonal.
- A prerequisite to create these sub-channels by beamforming at Tx and Rx is that both have knowledge about the channel



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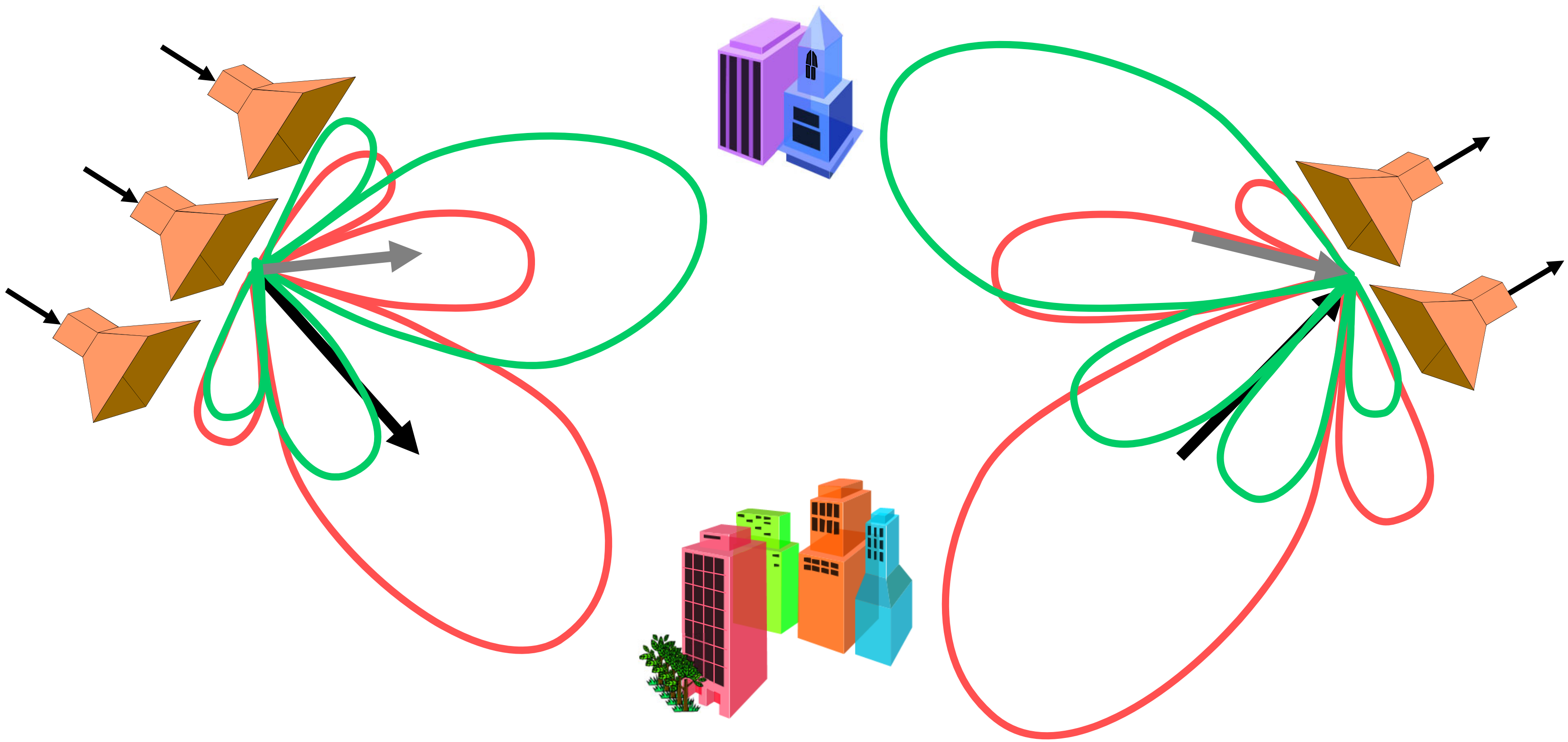
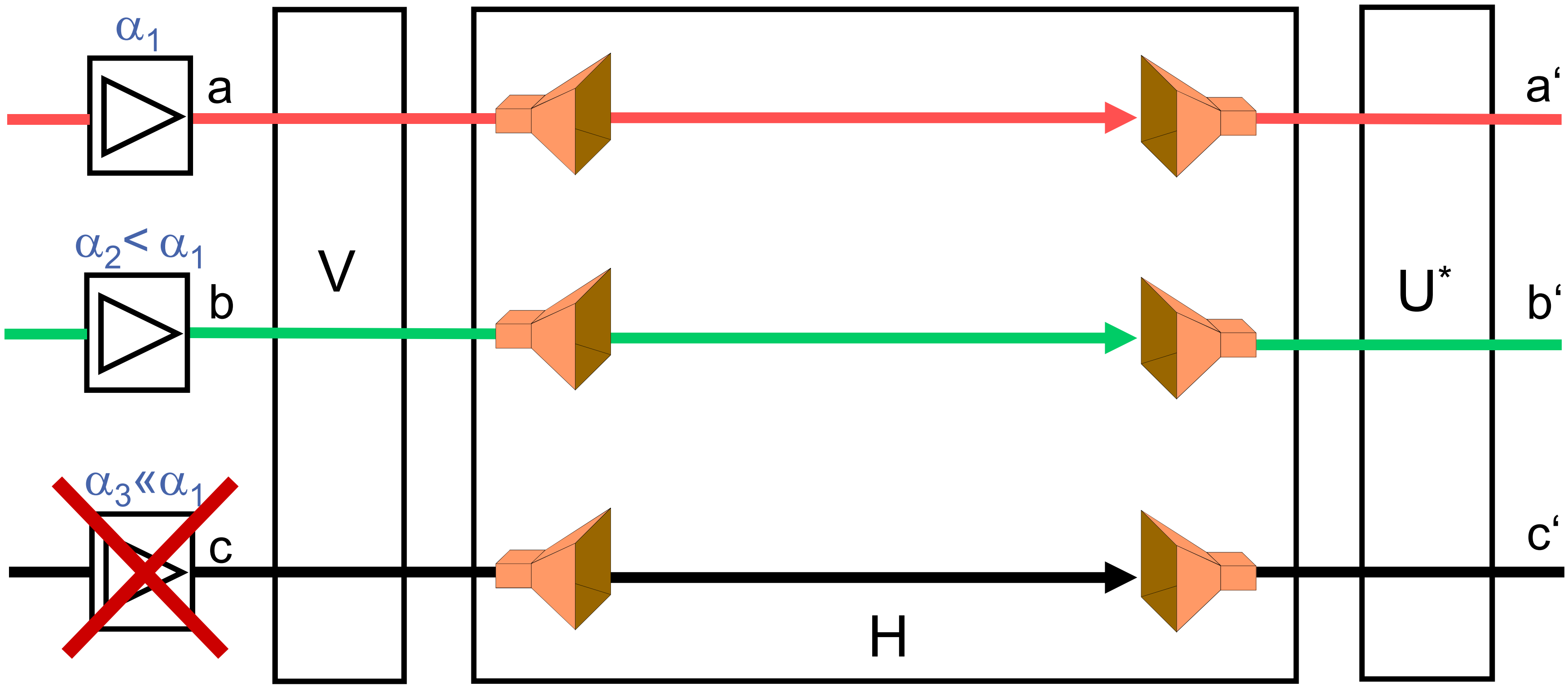
Sub-Channels



Power Control ⇒ Waterfilling

good sub-channels → high power
bad sub-channels → low power



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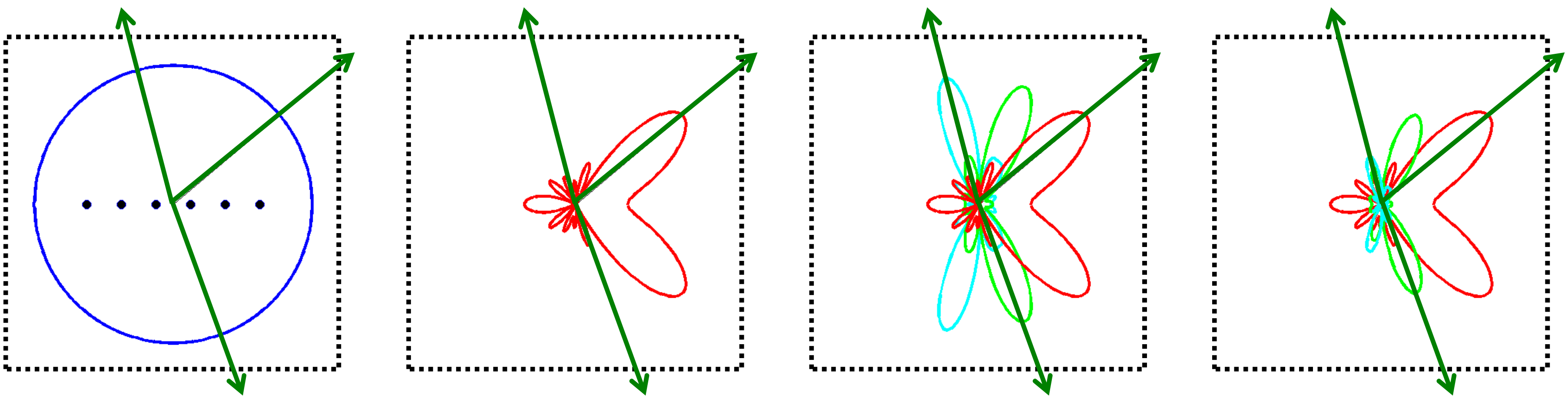
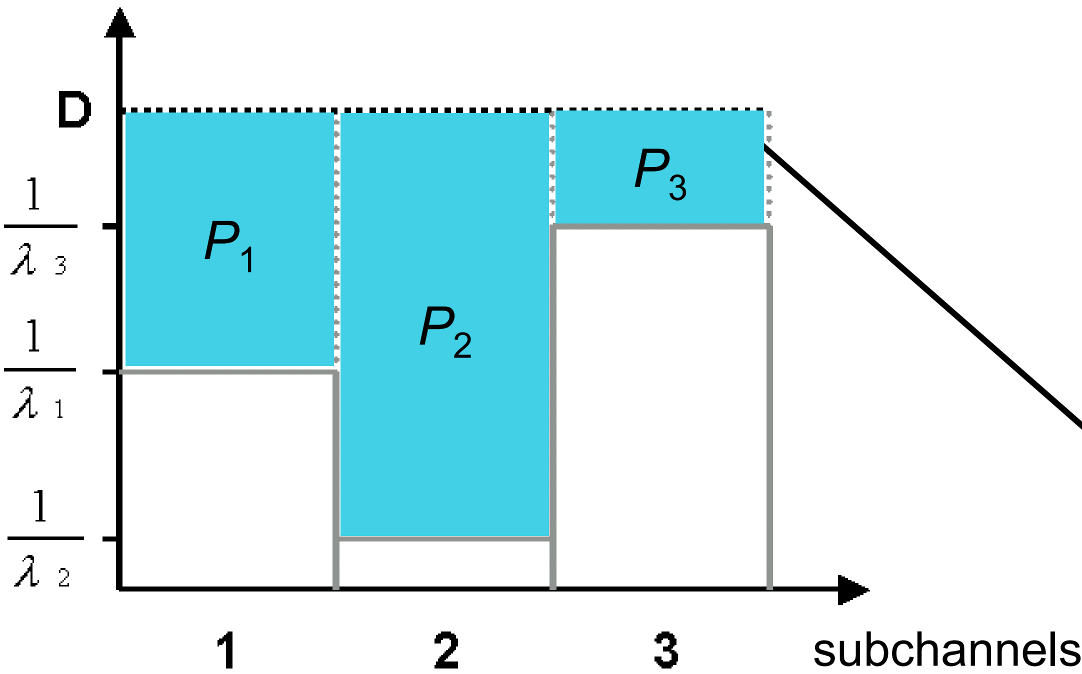
Distribution of Transmit Power

Uniform	Beamforming	Spatial Multiplexing **without** Waterfilling	Spatial Multiplexing **with** Waterfilling
all antennas with the same power	Best sub-channel with all power	Same Power P_{Tx}/M for all subchannels	Power distr. according to channel quality



1. Sub-channel — 2. Sub-channel — 3. Sub-channel — Uniform

Waterfilling



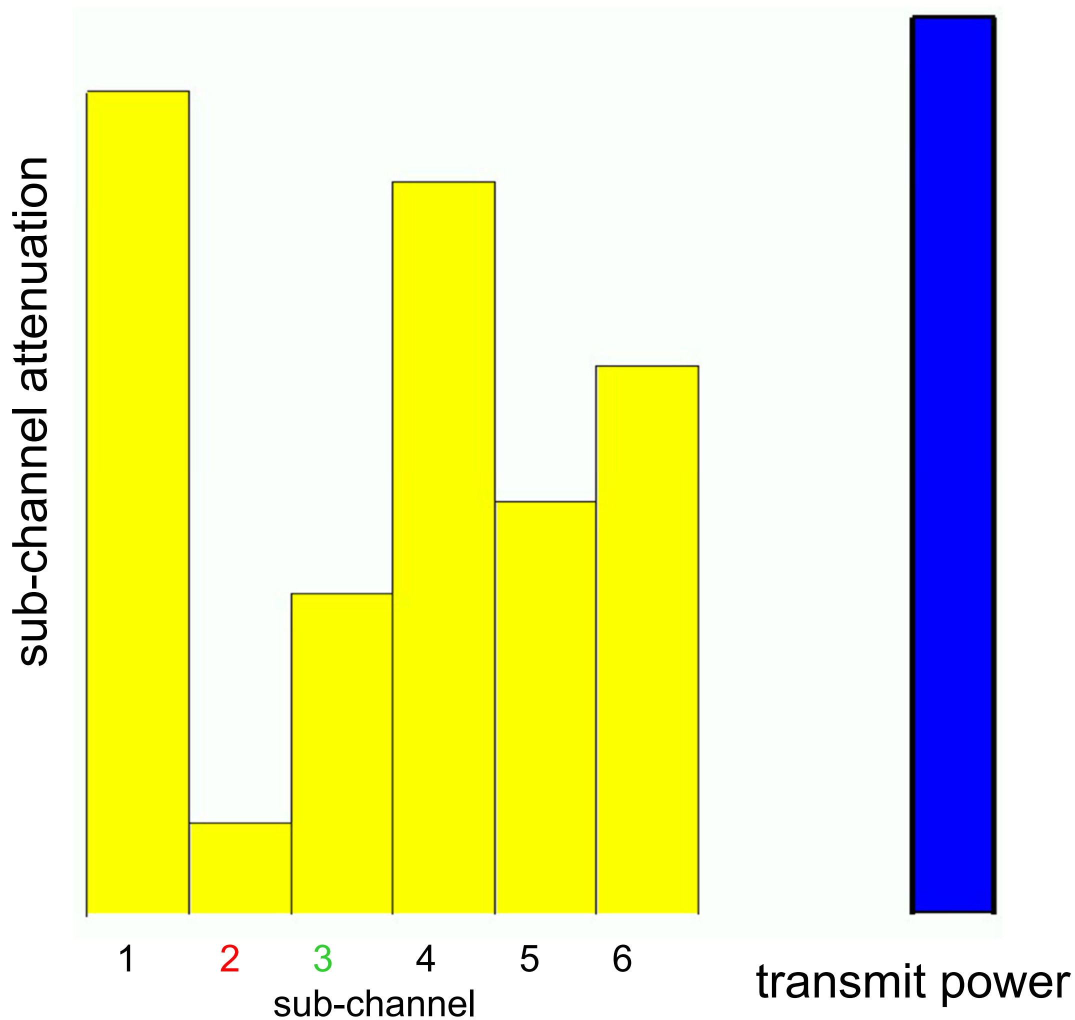
Power distribution depends on:
λ = eigenvalues{**HH***}

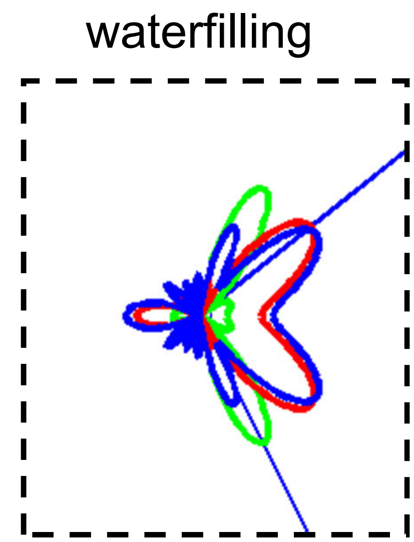
Threshold D:
$\sum P_m = P_{Total}$

Principle:

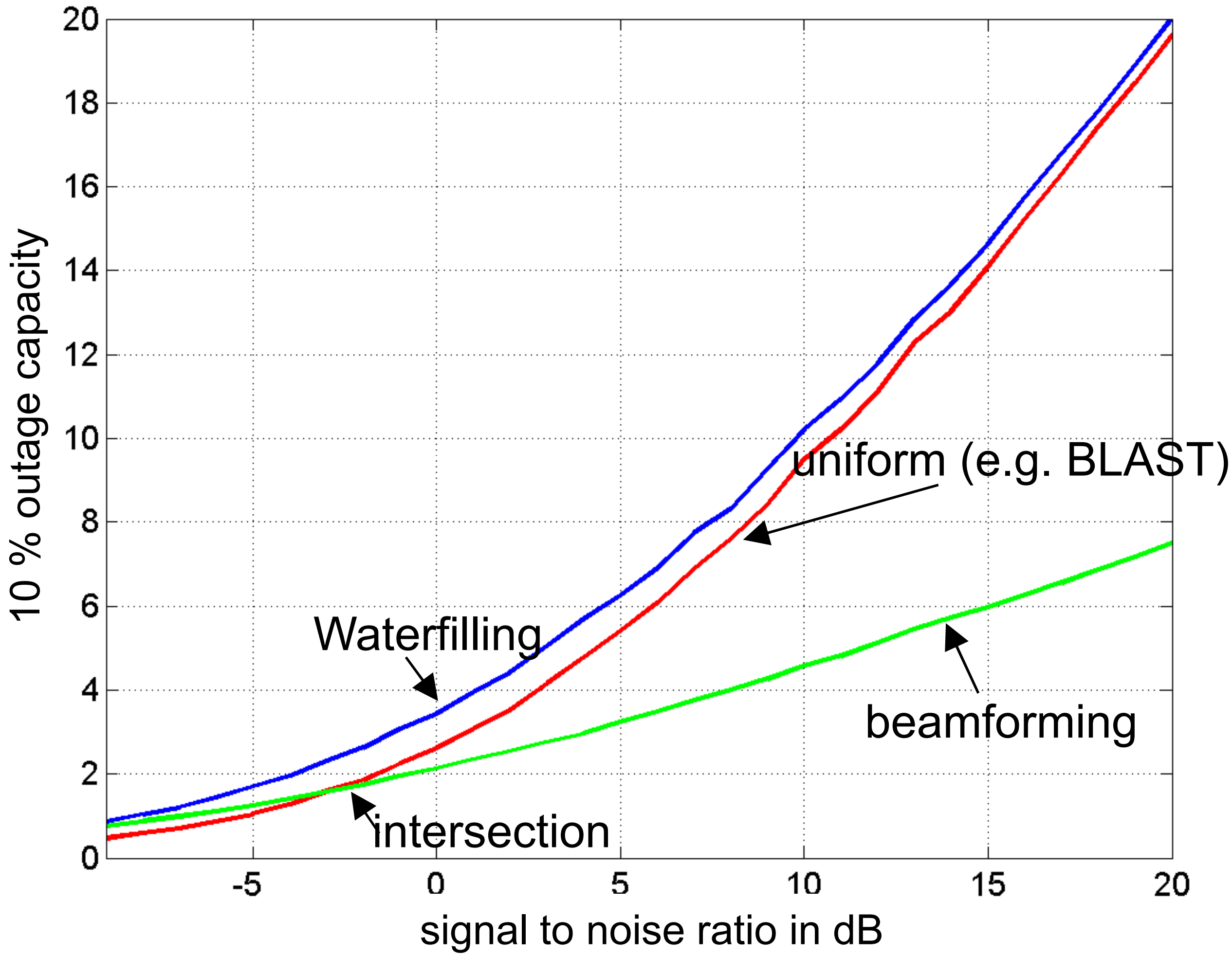
good subchannels → high power
bad subchannels → low power

Waterfilling

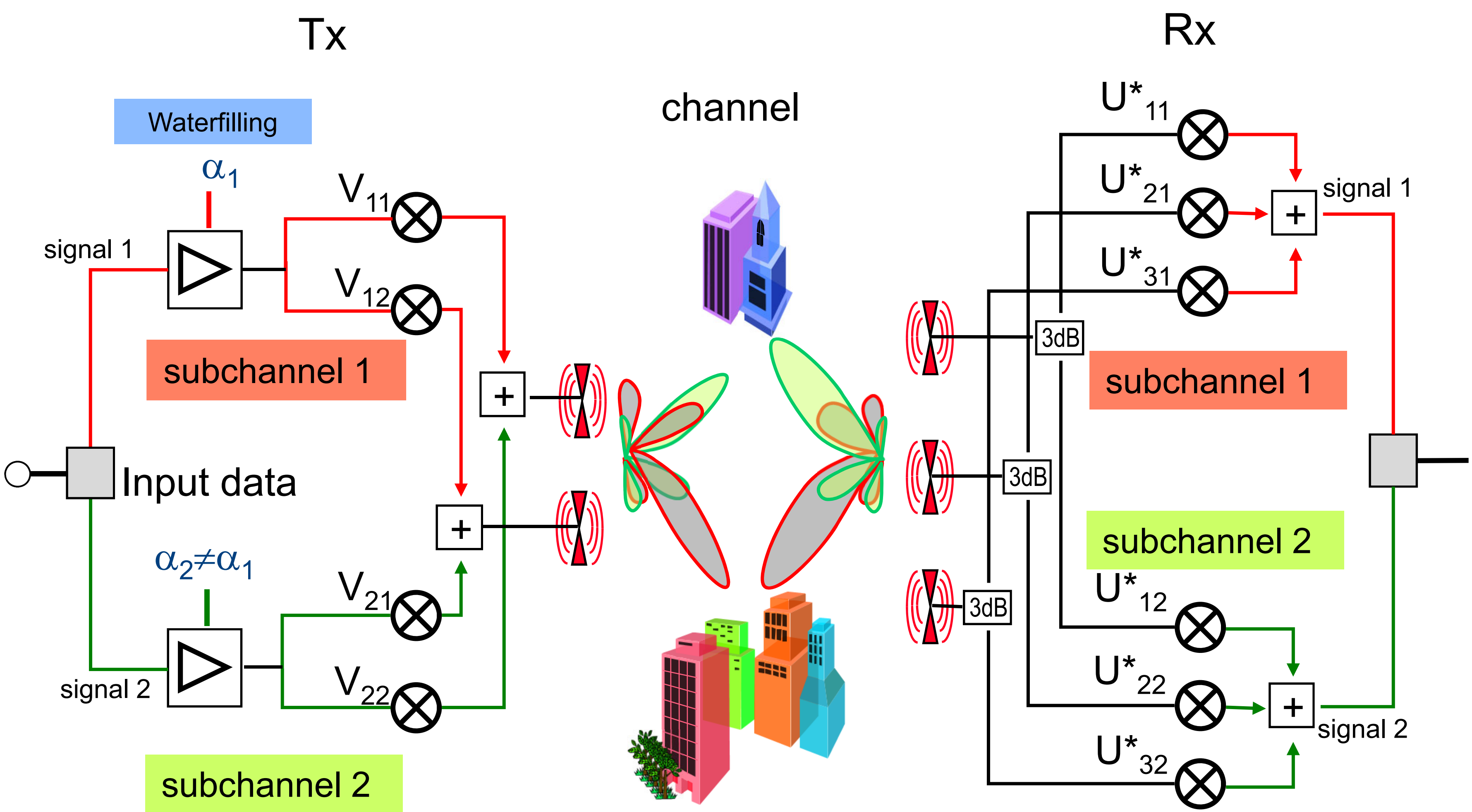




Comparison of Different MIMO Systems



2 x 3 MIMO Block Diagram



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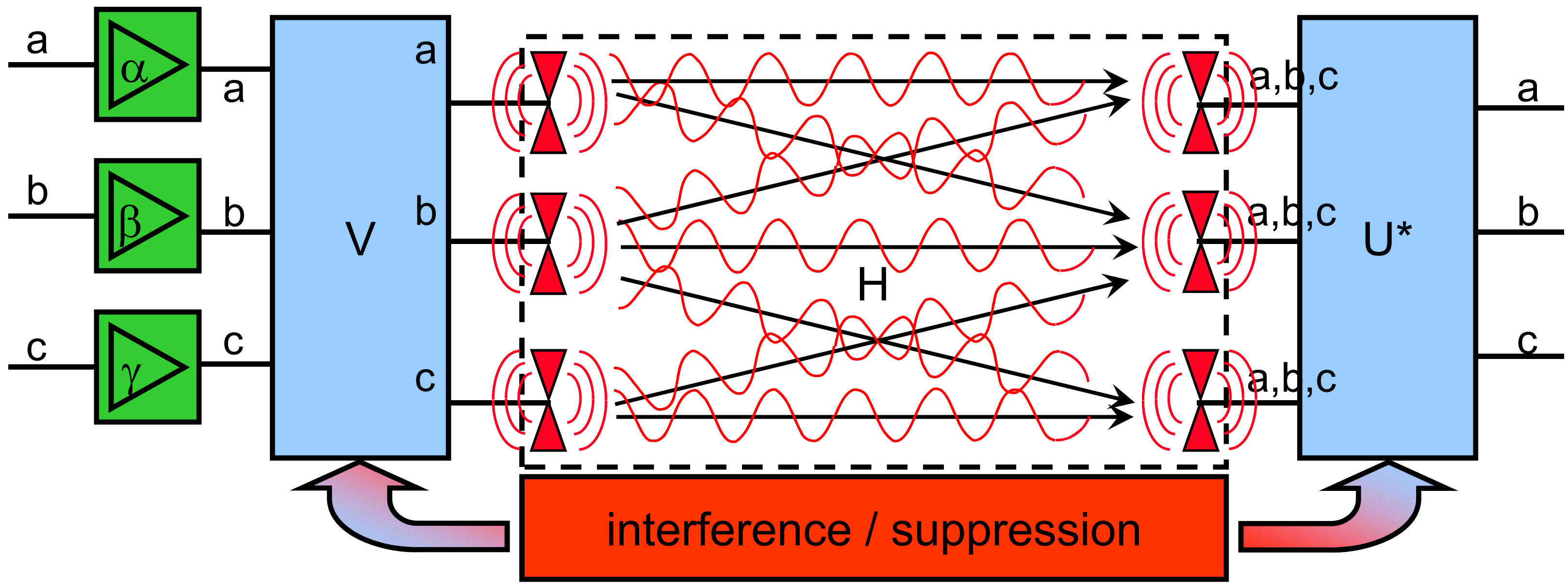
Options for MIMO Systems



sub-channel beam-forming

interference

waterfilling

$$C = \log_2\left[\det\left(\mathbf{I}_M + \mathbf{H}\mathbf{R}_{xx}\mathbf{H}^*\mathbf{R}_{zz}^{-1}\right)\right]$$